Bayesian learning in expert systems using plausible Petri nets

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Context

- Modern expert systems are dynamic, interactive, and designed to **automatically learn** from the contextual environment → Robots
- Petri Nets are the mathematical and computational formalism typically used to control robots and expert systems

MAIN FEATURES

1.- System level modeling
2.- Synchrony, concurrency, complexity
3.- Graphical language for interpretation
4.- Rely on mathematical principles
Basis of Petri Nets

- Bipartite directed graphs, composed by places (states) and transitions (processes)
- Places contain tokens that move to other connected place when transition is activated
- A token in a place is interpreted as holding the truth of the information represented by that place (e.g., "component failed")
- A transition is activated if all previous places are marked
- The distribution of tokens provides the state of the system (marking of the PN)

System dynamics is governed by state transition equation:

\[ M_{k+1} = M_k + A^T u_k \]
Basis of Petri Nets

Concurrency, Synchrony

![Petri Net Diagram]
Drawbacks for uncertainty representation

Formulated as a sequence of Booleans states: \{token, no token\} → \{true, false\} →
Insufficient for real-world (complex) systems

MAIN FEATURES

1.- System level modeling
2.- Synchrony, concurrency, complexity
3.- Graphical language for interpretation
4.- Rely on mathematical principles

DRAWBACKS

1.- Continuous knowledge representation
2.- Uncertainty
Petri nets variants

Limitation of most PN formalisms: Difficulty to rigorously deal with uncertain information, and to use such information to dynamically accommodate to the new contextual conditions.
Petri nets variants

Limitation of most PN formalisms: Difficulty to rigorously deal with uncertain information, and to use such information to dynamically accommodate to the new contextual conditions.
Plausible Petri nets\textsuperscript{1}

Mathematical features

- Two interactive subnets: \textit{numerical} (\(\mathcal{N}\)) & \textit{symbolic} (\(\mathcal{S}\))
- \(P^{(S)}\) contain a discrete amount of tokens;
- \(P^{(N)}\) contain \textbf{state of information} (degree of belief) about the state variable

\footnote{\textsuperscript{1}Chiachio et al, (2018). Information Sciences}
A new paradigm for uncertain knowledge representation by Plausible Petri nets

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ABSTRACT

This paper presents a new model for Petri nets (PNs) which combines PN principles with the foundations of information theory for uncertain knowledge representation. The resulting framework has been named Plausible Petri nets (PPNs). The main feature of PPNs resides in their efficiency to jointly consider the evolution of a discrete event system together with uncertain information about the system state using states of information. The paper overviews relevant concepts of information theory and uncertainty representation, and presents an algebraic method to formally consider the evolution of uncertain state variables within the PN dynamics. To illustrate some of the real-world challenges relating to uncertainty that can be handled using a PPN, an example of an expert system is provided, demonstrating how condition monitoring data and expert opinion can be modelled.

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PPNs: Information flow dynamics

**Definition:** After firing, the SI remaining in the preset is the *Conjunction of states of information* Conceptualizes the common (shared) information content
**PPNs: Information flow dynamics**

**Definition:** After firing, the SI in the postset is the *Disjunction of states of information.* It conceptualizes the aggregation of information content.
Execution dynamics: Example

The same result can be obtained through the equation (Chiachío et al. 2018):

\[
M^{(N)}_{k+1} = \left[M^{(N)}_k \circ \gamma_k + \left(\sum_{i=1}^{n_t} (a^+_{i})^T \otimes c_i + (A^-)^T \circ B\right) \cdot v_k\right] \circ \beta_k
\]
Bayesian learning using PPNs

PPNs have an inherent learning mechanism implemented in one of the elementary information flow operations (conjunction of states of information):

Assume that:

- \( f^{p_1} \) is a prior PDF \( p(x_k) \)
- \( f^{t_1} \) is a likelihood function \( p(D|x_k) \)

\[ f^{p_2} = \text{a posterior PDF } p(x_k|D): \]

\[ f^{p_1} \land f^{t_1}(x_k) \overset{\text{def}}{=} \alpha_{ij} \frac{f^{p_1}(x_k)f^{t_1}(x_k)}{\mu(x)} \propto p(D|x_k)p(x_k) \]
Illustrative example

Self-managed (intelligent) system for railway track maintenance

Activates maintenance actions based on updated uncertain information using CM data
Sequential Bayesian updating of track degradation

Observation uncertainty (Transition $t_1$)

$$p(y_{n+1}|z_{n+1}) = \mathcal{N}(y_{n+1}|x_{n+1}, \sigma_x)$$
Sequential Bayesian updating of track degradation

Updating: Conj. of states (place $p_1^{(\mathcal{N})}$)

$$p(x_{n+1}|y_{0:n+1}) \propto p(y_{n+1}|x_{n+1}) p(x_{n+1}|y_{0:n})$$
Results: Sequential estimation of permanent deformation
Results: System response after learning

- Operational rules (e.g., maintenance activities) change the "natural" evolution of the system
- Operational rules are autonomously and adaptively triggered based on the degree of belief of the system state
- System state is sequentially updated as long as further contextual changes (CM data) take place
Results: System response after learning

Token visiting scheme per symbolic place (in percentage)

- Working condition: 95.4% (100% for 2 data points, 100% for 5 data points)
- Warning State: 75.7%
- Inspections needed: 4.6%
- Undergoing inspections: 95.4%
Conclusions

- PPNs act as a hybrid system combining symbolic items and numerical values
- PPNs allow uncertain knowledge representation and learning from contextual information
- The learning capability appears naturally in one of the elementary operations for information flow

Future research

- Exploit their potential to model intelligent infrastructures
THANK YOU
Illustrative example

Section defined by:
- Homogeneous degradation mechanisms
- Track maintenance and renewal history

Condition assessed by:
- Experimentation
- Ballast fouling estimation
- Ground Penetrating Radar