Possibilistic Identification of Reliable Finite Impulse Response Models

Dominik Hose and Michael Hanss
Motivation

Modeling
- **White box**
  - First principles, laws of nature, ...
- **Black box**
  - Data-driven, statistical learning methods, ...

System Identification...
- discriminates between signal and noise.
- estimates “best” crisp dynamics.
- does not account for modeling deficiencies. 😞
  - If structure of the model does not match the true system structure.

Idea
- Incorporate all the visible dynamics in a fuzzy-valued model.
- Use this set of models for reliable predictions about the system.
Motivation

Fuzzy Set Theory in a nut shell

Finite Impulse Response Identification

Possibilistic Regression Analysis

Reliable Controller Synthesis
Fuzzy Sets

Perfectly known model parameter

Uncertain model parameter

\[ p = \tilde{p} \]

membership function \( \mu \)

crisp number \( \tilde{p} \)

fuzzy number \( \tilde{p} \)

"\( p \) is about \( \tilde{p} \)"

nominal value
Impulse response model of a continuous-time LTI system

- \[ y(t) = \int_{\tau=0}^{\infty} g(t - \tau)u(\tau) \]
  - \( g(t) \): impulse response

- Unique description of LTI system

  For non-linear systems it does not exist.

\[ \dot{x} = -x(x + 1) + u \]
Finite Impulse Response

- Discretized and Finite Approximation of Impulse Response
  \[ y[k] = \sum_{i=0}^{M} g[k - i]u[i] \]
  - Identification via Linear Regression Formulation

\[
\begin{bmatrix}
y[M]
\vdots
y[N]
\end{bmatrix}
\approx
\begin{bmatrix}
u[M]
\vdots
u[1]
u[1]
\vdots
\end{bmatrix}
\begin{bmatrix}
g[1]
\vdots
g[M]
\end{bmatrix}
\]

Least-Squares Estimate
\[ g^{LS} = (\Phi^T\Phi)^{-1}\Phi^Ty \]

- Optimal estimator if signal is perturbed by Gaussian noise.
Elementary Least Squares

\[
\begin{align*}
\min_{g^k} & \quad \| \Phi g^k - y \|_2^2 \\
\text{s.t.} & \quad y[k] = \sum_{i=0}^{M} g^k[k - i] u[i]
\end{align*}
\]

Solve for \( k = 1, \ldots, N \)
Constructing the fuzzy set

- Bounding box, convex hull, etc. of elementary coefficients

Possibilistic Interpretation: “How well does a specific parameter realization represent our system?”

Sensitivity information of least-squares problem is encoded in membership function.
The true non-linear system response is tightly and reliably approximated.
All the dynamics that we see in the data – but only those! – are accounted for in the prediction.
Find robustly stabilizing control law
\[ u[k] = -Cy[k] \]

Characteristic Polynomial
\[ \tilde{p}(z) = z^M + \sum_{i=0}^{M} \tilde{g}[i] z^{M-i} \]

How to evaluate uncertain stability event?
\[ S : \max |\tilde{\lambda}_i(C)| < 1 \]
Possibility measure

on the universe of discourse $\Omega$.

- $\text{Pos}(\Omega) = 1$,
- $\text{Pos}(\emptyset) = 0$,
- $\text{Pos}(\Theta) = \max_{\theta \in \Theta} \frac{\text{Pos}(\{\theta\})}{\pi(\theta)}$, $\forall \Theta \in \Omega$.

Dual necessity measure

- $\text{Nec}(\Theta) = 1 - \text{Pos}(\Omega \setminus \Theta)$, $\forall \Theta \in \Omega$.

Induced possibility distribution from fuzzy variable $\tilde{\lambda} : \Omega \to \mathbb{R}$ by fuzzy membership function

- $\text{Pos}(S) = \max_{\lambda : S(\lambda)} \mu_{\tilde{\lambda}}(\lambda)$.

$\Rightarrow \text{Pos}(S(C)) = \max_{\lambda : |\lambda| < 1} \mu_{\tilde{\lambda}}_{\text{max}}(\lambda)$
Fuzzy-valued finite impulse response model provides reliable predictions about (in-)stability.
Problem

- Classical system identification does not account for neglected dynamics.

Solution

- Identify a fuzzy-valued system model that incorporates all dynamics contained in the data.

Method

- Elementary Least Squares
- Minimizing Sets

Application

- Possibilistic System Analysis
  - E.g. Controller Synthesis

Thank you for your attention!