Optimising cargo loading and ship scheduling subject to uncertain sea levels

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Abstract

Until now, works in the field of tide routing (i.e. optimisation of cargo loading and ship scheduling decisions in tidal ports and shallow seas) have omitted the uncertainty of sea level predictions. However, from the widely used harmonic tide forecasts to the more performant hydrodynamic models, sea level predictions are not perfectly reliable. Consequences for the maritime industry are significant: current solutions to tide routing may be robust through the introduction of arbitrary slack but they are not optimal. Given the financial implications at stake for every additional centimetre of draft and the dramatic effects of a grounding, an investigation of tide routing from the perspective of risk analysis seems necessary.

Considering the journey of a bulk carrier between two ports, a shipping decision model is designed to compute optimal cargo loading and scheduling decisions, given the time series of the sea level point forecasts in these ports. The objective function is chosen from well-known risk-averse models. In a realistic case study between two British ports with 54-hour-ahead sea level predictions, four risk metrics are compared, as well as the effect of the underlying sea level stochastic representation. Results show the economic added-value of adopting a risk perspective in cargo loading and ship scheduling optimisation.

Keywords: Operational research in maritime industry; Robust optimisation; Decision-making under uncertainty

1 Introduction and literature review

1.1 Ship scheduling in tidal areas

A ship’s draft is the distance between the waterline and the bottom of the hull. It is a fundamental characteristic of a ship and forms a major constraint in terms of scheduling or cargo loading decisions because a poor choice can lead to grounding in tidal areas or shallow waters. Yet the research on ship loading has mostly focused on operations safety and logistic aspects (see for instance a review in [ ]). The question of scheduling with time-varying draft was not tackled until recently, when Kelareva and colleagues developed a deterministic procedure to optimise ship scheduling and cargo loading decisions of multiple vessels at a single port [ , ]. Their procedure is based on the short-term predictions of under-keel clearance provided by the DUKC® software (OMC International, 1993, described in [ ]). Similar operational tools have been developed since then, e.g. the MetOceanView solution, by the commercial brand of the New Zealand Meteorological Service). Specifically, from real-time environmental measurements, the physical responses to the ship moving in a dynamic environment (squat, heel and wave) are quantified in terms of their effect on the under-keel clearance. Recent and actual water depth measurements in navigation channels are integrated to tide predictions. All these factors are combined to determine the optimal cargo loading and ship scheduling decisions given estimations of the under-keel clearance. In such a deterministic optimisation approach, safety margins have to be introduced as the under-keel clearance is only estimated a priori. One can ask whether taking into account the stochastic nature of sea levels (and, consequently, the under-keel clearance) could reduce this safety margin to some theoretical minimum - this is one of the aspects investigated in the current paper.

The work of [ ] was extended to a shipping cost optimisation problem for a fleet considering time-varying draft restrictions at waypoints, variable ship speed and cargo loads as well as flow control through busy waterways [ ]. The specific waterway ship scheduling problem was later formulated by [ ], who integrated tide as a constraint in their approach to optimally schedule the flow of incoming and outgoing ships through different shipping channels (so that the waiting times were globally minimised).

Similarly, researchers focusing on the berth allocation problem, which aims at scheduling berth and crane allocation to optimise port throughput, introduced tide as a constraint only quite recently. While...
early works \cite{early1, early2} were more concerned with the quantification of the economic impact of tides on port operations, recent studies developed practical models and solutions for berth scheduling optimisation \cite{recent1, recent2} or quay crane allocation \cite{recent3} in tidal ports.

### 1.2 Shipping optimisation in stochastic environments

Maritime transportation is an activity particularly subject to risk, i.e. the possibility of a loss. From the weather at sea to port variables (berth availability, loading/unloading works), including the volatility of bunker fuel prices, a range of uncertain factors condition the outputs of a shipping operation. In spite of its significant impacts on shipping productivity, the issue of uncertainty has remained marginal in the research on maritime transportation until recently. Indeed, as stressed by \cite{uncertainty1}, due to the complexity and untractability of some shipping problems, authors often introduce simplifications (constant speed, single cargo type, basic weather model, etc.) that are different from one study to another, making comparison difficult. The introduction of stochasticity is often limited to the modelling of a single or a very limited number of factors (e.g. weather \cite{weather1}, market demand \cite{market1}, weather and berth occupation \cite{berth}).

Water depth is also a significant uncertain factor. Although tide forecasts used to predict the water depths in shallow seas are traditionally given by harmonic analysis from past observations, a range of causes can modulate the observed water levels. These encompass weather influence, river discharge, the interaction between currents, shallow water seabed and ship traffic \cite{currents1} and lead to significant deviations between astronomical tides and actual water level observations (called residuals hereafter: the difference between observations and predictions). \cite{residuals1} estimate that the root mean square error on the high tide predictions in UK tide stations is typically 10 cm and rises to 29 cm for high tidal range stations. \cite{range1} note that sea level residuals can amount to 30% of the total measured sea level in Hillarys Boat Harbour, Western Australia.

Uncertainty about future water depths has considerable impacts on shipping optimisation. First, as shown in the case study presented in Section 1.4, even for a small-sized carrier of horizontal dimensions 85 m ×15 m, one additional centimetre of under-keel clearance can be turned into an extra freight of about 13 metric tons (mt) whose value ranges from US$ 2,500 for a single hold of malting barley\footnote{London Metal Exchange market data, January 2018.} to more than US$ 223,000 for a single hold of tin\footnote{Agriculture & Horticulture Development Board, UK Prices, January 2018.} with little increase in operational costs in short journeys\footnote{Agriculture & Horticulture Development Board, UK Prices, January 2018.}. Secondly, when it costs thousands of dollars a day to operate the same vessel, missing a tide-window because of a negative anomaly in the water depth is significantly costly to the shipper, to say nothing about the cost of grounding and its potential environmental consequences.

### 1.3 Robustness in shipping optimisation

In all the approaches mentioned in Section 1.2, water depths are considered as perfectly predictable variables. Although \cite{approach1} introduced a conservative 15-minute departure window for each departure/arrival decision, the authors justified the slack as a way to take into account the inertia of large ships in port operations rather than to account for sea level uncertainties. Unfortunately, as actual water levels are often different from predicted, such a deterministic assumption is either not robust or requires the introduction of safety margins to make it robust, at the cost of optimality \cite{margin1}. Large operational costs of ships tend to prevent the shippers from adding significant slack in their schedule \cite{slack1}, a ship being productive only when it is sailing.

An original approach to robustness in ship routing and scheduling is found in \cite{approach2}, who introduced the concept of “risky arrival”. A penalty cost proportional to the risk of a given schedule is integrated when optimising the transportation cost of a fleet. The work of \cite{risky1} should also be mentioned as it questions the applicability of mathematical optimisation in a real port context. The authors especially highlight the situation where a small change in the model inputs leads to a radically different optimal solution. The concept of persistence is introduced as a new feature of the optimisation model, so that small changes in the input values do not drastically change the nature of the optimal solution.

### 1.4 Objective and contribution

The present work aims at filling a gap in the field of tide routing by considering the uncertainty associated with water depths. A robust analysis of cargo loading and ship scheduling decisions in tidal areas is drawn through a realistic case study. The question at hand is: how can we optimise the cargo loading and ship
scheduling decisions, given imperfect sea level forecasts, without foregoing safety? To this purpose, a simple 1 ship - 1 leg maritime shipping decision model is introduced. The model assumes that an industrial operator has the sea level forecasts at two ports at a given time $t_0$, over a prediction horizon $T$. On this basis, the operator has to decide the amount of a given commodity to convey from one port to the other and when to depart. This model does not address the uncertainty associated with the under-keel clearance arising from dynamical responses to the sea state (heeling, heaving, squat effect), nor from imperfect or changing bathymetry. We limit our point to the uncertainty about still water levels, resulting from deviations to the tide predictions. The dynamical sources of uncertainty could, however, be integrated into a similar approach in order to address the open water problem.

In the following, our model allows us to demonstrate the economic potential of robust under-keel clearance optimisation. Beyond the application to industrial shipping, for which the bulk cargo load is quite flexible, this work wants to raise awareness of the economic potential for small vessels (mini-bulkers), cheap commodities (grains) and small ports strongly affected by tidal effects (i.e. limited dredging). In the current context of transportation greening, we expect this to be an important area for future applications.

Section ?? has introduced the motivations for the investigation of robust cargo loading and scheduling optimisation in tidal areas and outlined the state of the art around this issue. Section ?? introduces the case study and sets up the economic shipping decision model. In Section ??, the uncertainty on port sea level forecasts is discussed and a selection of four risk metrics, providing so-called robust alternatives to the deterministic decision-making process are presented. The implementation setting is summarised and the statistical modelling of sea level residuals is introduced. Section ?? discusses the results of these approaches and assesses their distributional robustness over a range of sea level models. Finally, the findings are summarised in Section ?? and perspectives are opened.

2  Shipping decision model

2.1  Case study

To illustrate the approach in this paper, a case study is presented. This gives the reader context for the model development that is detailed later. We consider a farm cooperative that owns a small-size bulk carrier of design draft 5.2 m, dimensions 85 m $\times$ 15 m and carrying capacity 5,170 mt. The company uses it to carry various farm commodities between ports along the United Kingdom coast. The malting barley produced in the fields of Eastern England is much desired by breweries further south in the UK. Consequently a single hold of malting barley is regularly shipped from Lowestoft to Portsmouth.

Lowestoft inner harbour offers quays with a maintained depth of at least 5.7 m regardless of the tide [? ]. This allows the bulk carrier to arrive and berth at any time, provided it is not fully laden. However, in order to facilitate sustainable transport connections, the arrival and unloading take place at the Harbour Railway Jetty of Portsmouth port. The latter is a limiting factor for time scheduling and cargo loading because nautical charts indicate a depth of 2.3 m below Chart Datum [? ]. Given the range of tides in the Portsmouth site, this means that the water depth varies between about 2.4 and 7.6 m which restricts the access of the vessel to the jetty, even with empty tanks. Given the vessel size and a freight unit value of US$ 195.61 per metric ton$^4$, 1 cm of additional draft equals an extra freight of 13.05 mt which conveys an extra profit of US$ 2,556. As described before, although a heavier ship will consume more fuel, for small vessels and short sea voyages, it remains much more profitable for the company to increase the overall cargo loading if possible.

On November 19th 2016 at 16:30 UTC, we assume that the cooperative has to decide how much barley will be freighted and when the vessel will depart. To this purpose, they use the long term harmonic tide forecasts as sea level predictions. Hence the problem at hand: given the economic, vessel and port parameters summarised in Table ??, given the tide predictions in the tide gauge stations of Portsmouth Harbour and Lowestoft Port$^5$:

1. What is the optimal decision in terms of cargo load and departure time, if the harmonic tide forecasts were considered as perfect?

2. Is this decision robust to actual port and sea level conditions?

3. What is an optimal and robust shipping decision if the uncertainty on tide forecasts is taken into account?

4. What shipping benefit can be guaranteed, given an error level, from the robust solution?

$^4$Agriculture & Horticulture Development Board, UK Prices, January 2018

$^5$The tide predictions are relative to the Harbour Master’s Office gauge station in Lowestoft and to the Victory Jetty gauge station in Portsmouth. Both are maintained and provided by the UK Environmental Agency.
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Table 1: Model parameters.

<table>
<thead>
<tr>
<th>Type</th>
<th>Description</th>
<th>Lowestoft</th>
<th>Portsmouth</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Journey</td>
<td>Decision time</td>
<td>19-Nov-2016, 16:30:00</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Mean distance between departure and arrival ports</td>
<td>195</td>
<td>Nautical miles</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Sea water mean density</td>
<td>1.025</td>
<td>Kilogram per cubic meter</td>
<td></td>
</tr>
<tr>
<td>Ship design</td>
<td>Mean operational sailing speed</td>
<td>13</td>
<td>Knot</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Ship horizontal surface</td>
<td>15 × 85</td>
<td>Meter×Meter</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Minimum cargo load (ballast)</td>
<td>1,870</td>
<td>Metric ton</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Deadweight tonnage (carrying capacity)</td>
<td>5,170</td>
<td>Metric ton</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Half-laden ship draft</td>
<td>5.2</td>
<td>Meter</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Fuel consumption rate of the laden ship at sea</td>
<td>8</td>
<td>Ton per day</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Fuel consumption rate of the ship at port</td>
<td>1</td>
<td>Ton per day</td>
<td></td>
</tr>
<tr>
<td>Monetary</td>
<td>Fuel cost</td>
<td>387</td>
<td>US$ per ton</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Other operational costs (staff, maintenance)</td>
<td>2,500</td>
<td>US$ per day</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Average bulk cargo value</td>
<td>195.6</td>
<td>US$ per ton</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Berthing and loading/unloading operation cost within normal opening times</td>
<td>1,486</td>
<td>1,239</td>
<td>US$ per hour</td>
</tr>
<tr>
<td></td>
<td>Berthing and loading/unloading operation cost outside of normal opening times</td>
<td>1,858</td>
<td>1,548</td>
<td>US$ per hour</td>
</tr>
<tr>
<td></td>
<td>Daily port fee</td>
<td>1,363</td>
<td>1,115</td>
<td>US$ per day</td>
</tr>
<tr>
<td>Port</td>
<td>Bulk material (un)loading rate</td>
<td>1,000</td>
<td>1,200</td>
<td>Ton per hour</td>
</tr>
<tr>
<td></td>
<td>Normal port opening time</td>
<td>[7 : 00, 19 : 00]</td>
<td>[7 : 00, 19 : 00]</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Minimum allowed under-keel clearance to navigate in port still waters</td>
<td>10% static draft</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>Forecast</td>
<td>Sea level forecast time step</td>
<td>15</td>
<td>Minute</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Horizon of the sea level predictions</td>
<td>54</td>
<td>Hour</td>
<td></td>
</tr>
</tbody>
</table>

5. How do the robust solution and guaranteed benefit depend on the residuals modelling and risk metric under consideration?

2.2 Model overview

The model used here is a simplified low-dimensional representation of the maritime inventory routing problem. A material is produced at a given rate in a production site (called loading port) and consumed at other sites (called unloading ports) at specified rates. Given storage capacities in the production and consumption places, what is the optimal design of routes and fleet schedule to minimise the shipping costs (sailing and port costs) without interrupting any of the production or the consumption in the aforementioned sites? The optimisation is made on an industrial shipping basis. In other words, the shipper owns the material to be shipped and wants to maximise the net benefit of the shipment (the value of the cargo loaded minus the shipping costs). The fleet consists of a single bulk carrier or general cargo ship and the study is restricted to a leg between the loading (departure, \( p_1 \)) and unloading (arrival, \( p_2 \)) ports. A constant ship speed is assumed (provided by the ship specifications). From this, the goal is to optimise the decision vector \( d \), consisting of the departure time \( t_d \) and the cargo \( m \) to be loaded given the sea level predictions available at time \( t_0 \), spanning the horizon \( T \) in the entrance channels of both ports and given constraints from the ship design (carrying capacity), safety at sea (minimum acceptable water under keel), port management (opening times and price bands for port labour). For now, unlimited
storage capacities in both ports are assumed. The question of rate of production in the departure port (i.e. offer) and rate of consumption in the arrival port (i.e. demand) is not taken into account.

In this simplified problem, the ship is assumed to be in the departure port at time $t_0$ with empty tanks. The most recent predictions $\hat{X}_p(t)$ for the sea levels in both ports $p = \{p_1, p_2\}$ over the horizon $T$ are available. Time is discretised with the time step $\Delta t$ (following the precision in the sea level prediction and observation time series). Here and in the following, in order to simplify the notations, $t_d$ will be relative to the origin of our time axis, $t_0$.

2.3 Model description

The model takes time series of sea level point-forecasts in both departure and arrival ports as inputs. Given contextual parameters regarding the journey, including ship characteristics, freight and port management, generic constraints about acceptable under-keel clearance, latest arrival time and cargo load and finally the net return computation rule for a journey, it computes the optimal cargo loading and departure time by means of a particle swarm optimisation (PSO) solver.

2.3.1 Journey parameters

Table ?? defines the model’s input parameters. A few comments and justifications are provided here.

The operational speed is assumed to be fixed and constant over the journey (as it is often the case in maritime shipping models). Operational port costs are subject to price bands. Although most often docks and loading / unloading operations are accessible 24 hours a day 7 days a week, the cost of such operations depends on the local port schedule. For example, midweek vs weekend periods for Liverpool port are shown in [? ? ]. The simple price band framework allows us to simulate a range of situations: nights vs days, week days vs weekends, bank holidays. Finally, the safety margin coefficient, $\alpha$, in terms of legally required under-keel clearance to use the confined navigation channel of port $p$, is set to 10% of the laden ship draft as this is usual practice at limited speeds [? ]. The open sea version would require adding a 30% margin to the dynamical draft.

2.3.2 Sea level input variables

The sea level point predictions in each port are harmonic tide forecasts, available online through the British Oceanographic Data Center portal. The time step, $\Delta t = 15$ minutes, sets a minimum bound on the resolution of our departure time solution.

2.3.3 Model variables

The ship draft, a key element in shipping planning and realisation, is a function of the cargo load as well as the fuel mass in the tanks at the time of interest. The fuel mass is estimated from the fuel consumption rates at sea and at port, the time already spent at sea and at port respectively, as well as the total fuel load necessary to move the ship from one port to another and (un)load material. Considering Archimedes’ principle and the equilibrium of forces in a gravitational field, the draft can be estimated from the equality between ship’s weight and water displacement. The latter is a function of the half laden ship’s draft, the ship’s horizontal area, its carrying capacity and the water density. Dynamical effects such as the squat effect or the heel due to the wind and the wave responses can reduce the under-keel clearance temporarily. They are not taken into account here beyond the safety margins because, as stated previously, we consider the still water problem.

2.3.4 Constraints

The ship’s cargo and scheduling have to satisfy a few constraints. First, the cargo load $m$ cannot exceed the tank capacity and must fit with the requirements for safe structural behaviour of the hull (i.e. equal minimum ballast). $m$ includes the fuel load necessary to carry the laden ship on the mean distance between the two ports at specified speed and load/unload the freight at specified rates in each port. Second, to enter/leave a port at a given time, the water depth must be greater than the ship draft plus the safety margin. Third, the ship cannot leave port $p_1$ before the cargo is loaded and must arrive in $p_2$ before the horizon $T$ is reached. Finally the time of arrival (i.e. berthing at port $p_2$) must not exceed the prediction horizon $T$. 

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2.3.5 Shipping return

The problem is to find the optimal combination of decisions \( d^* = (t^*_d, m^*) \) that maximises the net benefit \( B \) (hereafter called objective function or objective), where:

\[
B(t_d, m; X_{p1}(t), X_{p2}(t)) = \begin{cases} 
V - (O + P + U) & \text{if delivered on time,} \\
Z & \text{otherwise.} 
\end{cases}
\]  

(1)

The gross value \( V \) is the merchant value of the cargo \( m \) unloaded in \( p_2 \). It depends on the unit value of the freight. From there, we subtract the operational costs of the journey, starting from \( t_0 \) (time of decision) with an empty ship and finishing after unloading the material in port \( p_2 \). These charges encompass the propulsion costs \( O \), computed as a function of the total time spent at sea and at port respectively, the fuel unit price and the fuel consumption rates at sea and at port respectively. Operational charges also include usage costs \( U \), depending on the total time of the journey, loading and unloading steps included, as well as on the hourly usage cost (staff) of the ship. Finally, port costs \( P \) have to be included. They are calculated on the basis of the daily port fee, hourly handling prices in normal hours and outside normal hours in each port as well as the time spent in each port within and outside normal hours respectively.

\( Z \) is the cost of not making the delivery in time (i.e. within the horizon \( T \)). Depending on the aim of the user, \( Z \) can also take into account the negative externalities on the environment and society of a grounding (\( Z \rightarrow -\infty \)) or simply the loss for the shipper (\( Z = -V - (O + P + U) \)).

3 A probabilistic approach to decision making

Using the model described above, one can choose an optimisation technique (e.g. particle swarm optimisation or simulated annealing) to compute the optimal decision to take at time \( t_0 \), according to the sea level forecast time series \( \hat{X}_p(t) \) for the two ports \( p = \{p_1, p_2\} \). Such a calculation does not consider the actual stochastic behaviour of the water depth. Mean sea levels are locally influenced by a range of factors, including weather. A residual \( e_p(t) = X_p(t) - \hat{X}_p(t) \) between the predictions and the observations can lead to either a regret (\( e_p > 0 \): the shipper could have loaded more or departed earlier) or a loss (\( e_p < 0 \): in order to adjust to the actual water level the journey is delayed, or a grounding can happen).

In other words, the resulting solution is risky as it does not tolerate a negative deviation to prediction nor port delays. In order to account for the uncertainty on the outcome of a given decision and its potentially dramatic consequences for the shipping company, it is sensible to work in the frame of risk averse optimisation.

From the classical mean-risk [?] and chance-constrained perspectives [?] to the more recent so-called robust optimisation models (e.g. worst-case, minimax regret, uncertainty sets, see ?) for an historical overview and ? for an extensive presentation), operational research has developed a range of approaches to address the notion of uncertain decision-making. In these problems, the questions at stake are: a) Are all the scenarios acceptable, or feasible, whatever their probability of occurrence? (e.g. is a ship grounding acceptable?) b) How much does the decision-maker give way to objective optimality in order to guarantee feasibility? Any solution to stochastic optimisation is a trade-off between feasibility and performance, or said otherwise, between variance and guaranteed value of the objective function.

A (robust) optimisation approach must thus define the attitude of the decision-maker towards risk and the specificities of her optimisation problem before computing any solution. Between the two extreme approaches that are worst-case (always feasible) and deterministic optimisation (best performance e.g. for the most probable scenario, no uncertainty taken into account), lie a range of models depending on the decision-maker’s requests as regards performance and feasibility. We introduce in the following a representative selection of them, before comparing their outputs in Section ??.

3.1 Risk models
3.1.1 Regret

In decision-making under uncertainty, it is common to adopt the gain shortfall perspective. In this case, risk takes the meaning of the loss in profit due to the fact that decision \( d \in D \) is taken at time \( t_0 \) based on imperfect forecasts \( \hat{X}_p \in X \) of the environment state \( X_p \in X \). Let \( F_p \) be the cumulative distribution function over \( X_p \), which is conditional on information on the prior values of \( X_p \) and possible other information. Let \( \hat{F}_p \) be a predictive distribution of \( X_p \) (that is a distribution over \( \hat{X}_p \)) provided by the forecaster at \( t_0 \). Let \( \hat{X}_p(t) \) be a point forecast time series of \( X_p(t) \) over time \( [t_0, t_0 + T] \). \( B(\cdot) : D \times X \rightarrow \mathbb{R} \) the utility function (namely the net benefit of the journey based on decision \( d \)) and \( g(\cdot) : X \rightarrow D \) an
optimal action function defined by:

\[
y(\hat{F}_p) = \arg \max_{d \in D} \left( \mathbb{E}[B(d, \hat{X}_p)|\hat{F}_p] \right) = \arg \max_{d \in D} \int_{\mathcal{X}} B(d, \hat{X}_p)d\hat{F}_p
\]  

(2)

The loss function \(L(.,.) : D \times [0, 1] \rightarrow \mathbb{R}\) is then defined by as:

\[
L\left( y(\hat{F}_p), F_p \right) = B\left( y(X_p), X_p \right) - B\left( y(\hat{F}_p), X_p \right)
\]  

(3)

for all \(\hat{X}_p, X_p \in \mathcal{X}\). In other words, the utility of the decision made under uncertainty \(B\left( y(\hat{F}_p), X_p \right)\) is compared to the utility resulting from the decision made under perfect knowledge of the future \(B\left( y(X_p), X_p \right)\).

With an absolute robust approach, each possible shipping decision \(d\) is mapped to the maximum loss it can generate, whatever its probability of occurrence. The optimal decision minimises:

\[
d^* = \min_{d \in D} \left\{ \max_{F_p} \left\{ L\left( d, X_p \right) \right\} \right\}
\]  

(4)

Its less conservative counterpart involves mapping each decision \(d\) to the regret it generates in average:

\[
d^* = \min_{d \in D} \left\{ \mathbb{E}\left[ L\left( d, X_p \right) \right]_{F_p} \right\}
\]  

(5)

Looking more closely at the definition of the loss which we aim to minimise (the expectation over the space of sea level residuals), one can notice that minimising \(\mathbb{E}\left[ L\left( d, X_p \right) \right]_{F_p}\) is equivalent to finding the decision \(d^*\) that maximises the expected benefit \(\mathbb{E}\left[ B\left( d, X_p \right) \right]_{F_p}\).

### 3.1.2 Mean-risk

What appears to be the first risk model developed in operational research involves adding a penalty known as the risk functional to the expected objective outcome of a given decision, and thus setting:

\[
d^* = \max_{d \in D} \left\{ \mathbb{E}\left[ B\left( d, X_p \right) \right]_{F_p} - \beta \mathbb{D}[B]_{F_p} \right\}
\]  

(6)

where the parameter \(\beta \geq 0\) allows to quantify the price of risk.

In the simplest case, the risk functional is proportional to the standard deviation of the objective:

\[
\mathbb{D}[B] = \left( \mathbb{E}\left[ (B - \mathbb{E}[B])^2 \right]_{F_p} \right)^{1/2}
\]  

(7)

Negative and positive deviations to the mean do not have the same implications in terms of risk. In the case of maximising the shipping benefit, positive deviations to the expected benefit are welcome, contrary to negative ones. The standard deviation cannot fully describe such asymmetrical behaviour of the utility function. The lower semi-deviation of order \(\gamma\) is consequently introduced as:

\[
\mathbb{D}[B] = \left( \mathbb{E}\left[ (B - \mathbb{E}[B])^\gamma \right]_{F_p} \right)^{1/\gamma}
\]  

(8)

Note that, in the following, we use \(\gamma = 2\) and \(\beta = 1\).

### 3.1.3 Worst-case

The absolute robust way of optimising the shipping net benefit is to prevent any unfeasible scenario and maximise the outcome in the worst possible scenario. In other words, finding the decision:

\[
d^* = \max_{d \in D} \left\{ \min_{F_p} \left\{ B\left( d, X_p \right) \right\} \right\}
\]  

(9)
3.1.4 Chance-constrained

Although strictly speaking robust in terms of feasibility, the worst-case approach is often criticised for being too conservative in practical implementations.

The chance-constrained perspective allows more flexibility. Given a level of guarantee $\zeta$, it computes the decision maximising the ensured benefit at this level, in other words:

$$d^* = \max_{d \in D} \left\{ \inf_{b} \left\{ P \left( B \left( d, X_p \right) \leq b \right) \leq 1 - \zeta \right\} \right\}$$

In our experiments, we use $\zeta = 0.99$, that is we look for the maximal benefit allowing an error rate less than or equal to 1%.

3.2 Implementation

The problem of deterministic shipping optimisation was defined in Section ???. It consists of finding the decision $d^* = (n^*, m^*)$ maximising the net benefit of the shipping given sea level forecasts $\hat{X}_p, p = \{p_1, p_2\}$. Similarly, the risk minimisation problem consists of finding the decision maximising one of the objective functions (or risk functions) defined in Section ???.

Both are constrained 2-dimensional optimisation tasks whose objective functions are not continuous nor differentiable. As a result, classical analytical optimisation techniques cannot be used. Hence the call to derivative-free algorithms such as particle swarm optimisation to estimate $d^*$. A range of other computational methods could have been implemented as well. However, PSO was chosen because it generally demonstrates good convergence and execution speed properties in addition to its simplicity of implementation. A review and comparison of the derivative-free approaches is provided in [? ]. PSO is an iterative stochastic optimisation technique that imitates the natural swarm behaviour of a bird flock [? ]. At each iteration, the elements (particles) of the flock explore the search space in a semi-random way and evaluate the fitness (value of the function to optimise) of their positions. They share the information so that their next move is influenced by both their own findings and the findings of the other members of the swarm. The algorithm stops when the desired number of iterations is reached and the position with optimum fitness is returned. Algorithm ?? describes the procedure and our implementation choices.

Because the risk functions defined in Section ?? cannot be written in closed forms due to the definition of the net benefit $B$, it is natural to turn to Monte Carlo simulations to estimate them, within the PSO procedure. Algorithm ?? shows the general approach, now referred to as $R_{PSO}$. $B_{PSO}$ refers to the "deterministic" optimisation of the shipping benefit (by means of Algorithm ??), that is without taking into account any uncertainty on the sea level forecasts (although technically PSO is a stochastic technique). Hereafter we name nominal state the forecasted sea-level state.

3.3 Distributional robustness

Mathematically, the risk functions defined above are based on the distribution of the economic output of a given decision when the sea levels in both departure and arrival ports vary around their nominal state. Such a definition implies that the risk evaluation is model-dependent: its accuracy depends on the quality of the modelling of sea level residual distributions. In this section, we summarise the results of an analysis of the residuals distribution in Portsmouth and Lowestoft ports.

The dataset used for the modelling and then the testing consists of sea level residuals sampled every 15 minutes between 00:15-01/01/2006 and 23:45-31/12/2016 UTC, in each port. We split it into two parts: even years (dataset $D_e$) and uneven years ($D_u$). The former is used for modelling the sea level residuals by means of best-fit distributions. It is then used for the shipping optimisation procedure per se. Finally, $D_u$ is used as validation set, to perform simulations and gather statistics on the performance of the optimisation outputs.

Three distributions are tested on $D_e$: Gaussian, Logistic and Gaussian mixture model (GMM). The number of components in the Gaussian mixture models (3 for Portsmouth, 5 for Lowestoft) were chosen so as to minimise the Akaike Information Criterion [? ]. This criterion assesses the “goodness of fit” of a model to a dataset while introducing a penalty that increases with the number of free parameters requiring estimation. The aim is to find the optimal trade-off between model complexity and loss of information.

Kolmogorov-Smirnov statistics were computed to quantify the “goodness-of-fit” of each model. They reject at the 1% significance level the null hypothesis that the residuals follow a Gaussian or Logistic distribution for both ports. A graphical analysis of the three models shows that the Gaussian distribution significantly under-represents the small deviations of sea level observations with respect to tide
**Algorithm 1** Particle Swarm Optimisation procedure

1. Initialise randomly the position $d_i$ of each particle $i$ in the search space $D$ and set their initial velocity vector to $0$.

2. For each step $j$:
   
   (a) For each particle $i$:
       
       i. Compute the objective function $f(d_i)$ (e.g., the net benefit $B(d_i, X_{p_1}(t), X_{p_2}(t))$) resulting from the shipping decision $d_i$ with actual sea levels $X_p(t)$, $p = \{p_1, p_2\}$. This is the fitness of position $d_i$.
       
       ii. Update the personal best $b_i(j)$ of each particle $i$, i.e., its position with optimal fitness among the set of previous iterations. Similarly, identify and update the global best $g(j)$, that is the best solution among the positions visited by the whole swarm so far.
       
       iii. Move each particle according to the following equation of motion:
            
            $$d_i(j+1) = d_i(j) + \nu_i(j+1),$$
            
            (11)

            where the velocity is defined by:
            
            $$\nu_i(j+1) = \omega(j+1)\nu_i(j) + c_1 R_1 \left(b_i(j) - x_i(j)\right) + c_2 R_2 \left(g(j) - x_i(j)\right).$$
            
            (12)

            The cognitive $c_1$ and social $c_2$ coefficients are set up so as to optimise the ratio between individual exploitation and social interaction while the linearly decreasing inertia weight $\omega(j)$ limits 'velocity explosion'. The diagonal matrices $R_1$ and $R_2$ introduce stochasticity in the walk of the particles. Their diagonal elements are generated from a Lozi 2-dimensional chaotic map [?].

3. Stop when the desired number of steps is reached.

**Algorithm 2** Procedure $R_{PSO}$

1. Initialise randomly the position $d_i$ of each particle $i$ in the search space $D$ and set their initial velocity vector to $0$.

2. For each step $j$:
   
   (a) For each particle $i$:
       
       i. Sample time series of tide residuals $e_p(t)$ in ports $p = \{p_1, p_2\}$ from a distribution model. In this first study, we assume the residuals to be independent from port to port at a given time, and between any two given times at a given port. The spatial independence is not a strong assumption as long as ports are not too close. On the other side, the time independence can be discussed as on short durations the residuals show correlation.
       
       ii. Compute the net benefit $B(d_i, X_{p_1}(t), X_{p_2}(t))$ for the simulated sea level conditions. These are given by the nominal state modified by the tide residuals, namely at port $p$: $X_p(t) = \hat{X}_p(t) + e_p(t)$.
       
       iii. Repeat steps ?? to ?? until the number $N_s$ of simulated environments requested to compute the empirical risk function is reached.
       
       iv. Estimate the latter from the $N_s$ outputs of step ??, e.g., for the chance-constrained approach at error level $1 - \zeta$, find the quantile of level $1 - \zeta$ (that is the cutoff benefit $b$ such as $P(B < b) = 1 - \zeta$ i.e. $P(B > b) = \zeta$).

   (b) Move particles according to the general PSO procedure described in Algorithm ??, in the search space, step by step. Here the objective function to be maximised are the risk functions computed in step ??, e.g., the cutoff benefit $b$.

   (c) Stop when the desired number of steps is reached and return the position with optimal fitness.
predictions. Hence the introduction of the Gaussian mixture model, that globally represents the original residual distribution with greater fidelity. Besides, the GMM is able to capture the long tails that the single Gaussian or Logistic cannot. This could be important, as extreme events are usually in the tails.

This analysis will first be used to assess the distributional robustness of the optimisation procedures in Section ??, by analysing the effect of the residual modelling on the optimisation results. Besides, on a standard desktop computer running Linux, sampling from a Logistic distribution is about 10 times quicker than from a Gaussian distribution and 15 times quicker than from a 5 component mixture distribution.

In the following sections we check whether the difference in risk outputs and its implication in real-world decision making justify the added complexity of the GMM input model.

### 4 Results & Discussion

All the results in terms of benefit $B$ will be expressed as multiples of the value of the minimum cargo load, $B_0 = \$363,550$. We also set the cost of not making the delivery in time to $Z = -V - (O + P + U)$. Negative benefits would thus imply a grounding or the impossibility to reach the arrival port within the specified time horizon.

#### 4.1 Deterministic case

The $B_{PSO}$ procedure recommends the ship to leave Lowestoft Harbour at 23:00 UTC on November 19th 2016 with an overall barley freight of 3,835.0 mt. The standard deviations of these recommendations are estimated to be 0.5 mt in freight and less than 15 minutes in time (from 1,000 independent runs).

Figure 1 presents a mapping of the final shipping benefit over the decision search space $D$, given the forecast $a$ priori at hand and given perfect forecasts, i.e. the $a$ posteriori exact observations of the sea level depths. The optimal decision according to $B_{PSO}$ in each scenario differ by 1 hour and 30 minutes in time and 527 mt in cargo load. In other words, the deterministic solution under imperfect harmonic predictions is far away from optimality in the real-world of non-zero residuals. Besides, it is quite straightforward to see on these maps that both solutions are very sensitive to perturbations. A 15 min departure/arrival shift or a negative error in the actual sea levels both shift the expected benefit from its maximum to the negative area.

One way to get over the second limitation is to improve the accuracy of sea level forecasts. This is currently achieved by means of storm surge models. To take into account the local weather perturbations, these models use atmospheric forecasts as forcing in shallow-water hydrodynamic simulations e.g. the CS3 storm surge model covering the sea of the northwest European continental shelf [??]. Nevertheless, whatever the accuracy reached, these forecasts cannot prevent the issue of port perturbations and delays. Hence it seems reasonable to develop a robust solution instead of a single deterministic optimisation.

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**Figure 1:** Mapping of the net benefit $B$ over all the decisions $(t_A, m)$ of the search space, given sea level forecasts at hand ?? or actual sea level ???. The optimal decisions based on the deterministic forecasts and on the perfect forecasts (i.e. real state of the sea) through the solver $B_{PSO}$ are also reported.
Table 2: Statistics over 50 runs of the outputs in terms of decision-making. The optimal cargo load $m$, departure time $t_d$ and guaranteed benefit $B_{98}$ at the level of 2% (over 100,000 simulations) are expressed in metric tons, UTC and fraction of $B_0$ respectively. The uncertainty is computed as the standard deviation of the results.

<table>
<thead>
<tr>
<th>Distribution Risk metric</th>
<th>GMM</th>
<th>Logistic</th>
<th>Gaussian</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean-Regret</td>
<td>$m = 3,947 \pm 20$</td>
<td>$m = 3,940 \pm 10$</td>
<td>$m = 3,957 \pm 9$</td>
</tr>
<tr>
<td></td>
<td>$t_d = 00:45 \pm 30mn$</td>
<td>$t_d = 01:00 \pm 15mn$</td>
<td>$t_d = 00:30 \pm 15mn$</td>
</tr>
<tr>
<td></td>
<td>$B_{98} = 1.901$</td>
<td>$B_{98} = 1.894$</td>
<td>$B_{98} = 1.909$</td>
</tr>
<tr>
<td>Worst-Case</td>
<td>$m = 3,943 \pm 15$</td>
<td>$m = 3,935 \pm 10$</td>
<td>$m = 3,961 \pm 11$</td>
</tr>
<tr>
<td></td>
<td>$t_d = 00:45 \pm 15mn$</td>
<td>$t_d = 01:00 \pm 30mn$</td>
<td>$t_d = 00:45 \pm 30mn$</td>
</tr>
<tr>
<td></td>
<td>$B_{98} = 1.899$</td>
<td>$B_{98} = 1.891$</td>
<td>$B_{98} = 1.908$</td>
</tr>
<tr>
<td>Mean-Risk</td>
<td>$m = 3,946 \pm 18$</td>
<td>$m = 3,933 \pm 15$</td>
<td>$m = 3,963 \pm 16$</td>
</tr>
<tr>
<td></td>
<td>$t_d = 00:45 \pm 30mn$</td>
<td>$t_d = 00:45 \pm 15mn$</td>
<td>$t_d = 00:00 \pm 30mn$</td>
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<tr>
<td></td>
<td>$B_{98} = 1.901$</td>
<td>$B_{98} = 1.895$</td>
<td>$B_{98} = 1.905$</td>
</tr>
<tr>
<td>Chance-Constrained</td>
<td>$m = 3,959 \pm 11$</td>
<td>$m = 3,956 \pm 9$</td>
<td>$m = 3,976 \pm 6$</td>
</tr>
<tr>
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<td>$t_d = 00:30 \pm 15mn$</td>
<td>$t_d = 00:45 \pm 15mn$</td>
<td>$t_d = 00:45 \pm 15mn$</td>
</tr>
<tr>
<td></td>
<td>$B_{98} = -2.239$</td>
<td>$B_{98} = 1.905$</td>
<td>$B_{98} = -2.253$</td>
</tr>
</tbody>
</table>

4.2 Risk models

We now use $R_{PSO}$ to compute the optimal shipping decision under uncertain sea levels. Each of the four risk metrics presented in Section ?? is combined with one of the three sea level residuals distribution models under consideration. Table ?? reports the statistical results of each combination as regards the optimal cargo load, departure time and the resulting guaranteed benefit at the error rate of 2%, that is the 2% percentile $B_{98}$. The latter is estimated from 100,000 Monte Carlo simulations. In order to prevent a methodological bias, these simulations sample the sea level by means of bootstrapping (over dataset $D_a$, c.f. Section ??).

As the purpose of the $R_{PSO}$ procedure is to support decision-making, it is necessary to analyse the consequences of the above results as regards their translation in terms of practical shipping decision. The overall majority of the computed departure times are located within a 30 mn time slot centered on 00:45. Taking into account the relative inertia of large vessels and generally slow port dynamics (from decision to subsequent actions), this range of uncertainty can be seen as a buffer to consider in the decision-making schedule. Trying to increase the precision on $t_d$ would be meaningless considering the real world context of a maritime shipping problem.

As expected, the worst-case approach is the more conservative and generally computes the lowest loads ($m \approx 3,945$ mt overall). The mean-risk model with a penalty equal to one standard lower deviation behaves very similarly. The chance-constrained approach returns the highest loads ($m \approx 3,966$ mt overall) and the mean-regret (or expected-benefit) approach is intermediary. This is a quite general observation, whatever the sampling distribution. As regards the distribution impact, Logistic sampling produces more conservative loads than the GMM approach and further again, than the Gaussian one. The difference between the maximal and minimal loads abovementioned is in the range of 30 mt, that is in our case study less than 3 centimetres of draft. This invariably leads to quite similar guaranteed benefits $B_{98}$ between the worst-case, mean-risk and mean-regret for a given distribution. On the contrary, the guaranteed benefit of the chance-constrained solution is much less stable: either maximum or minimum (and) negative. This illustrates the concept of distributional (non-) robustness: according to the sea level modelling (Logistic versus Gaussian and GMM), the solution computed by $R_{PSO}$ leads to either very satisfying outcomes overall or to a very likely failure.

Figure ?? summarises most of the information discussed above: whatever the risk metric, a Logistic sampling will produce more stable (smaller variance) outcomes than the other models. It also shows that, strictly speaking, only the mean-risk approach could be said to be distributionally robust. Indeed, the ranges of the reduction in standard deviation and in guaranteed benefit when the underlying distribution varies (2 and 0.6 % respectively) are much smaller than for the other metrics (closer to 8 and 0.9 % respectively). Considering the money at stake, even variations of 0.1% $B_{98}$ are worth a few thousand dollars, so should not be neglected. Three observations can be highlighted as well. First, in this particular case study, the stochastic optimisation based on risk metrics allows the owner to (in most of the configurations) save money as the guaranteed benefit is above the expected benefit of the deterministic
Figure 2: Performance of each optimisation approach (a risk metric combined with a sea level residuals distribution) from the perspective of the reduction of the guaranteed benefit at the error level of 2% and the standard deviation of the actual shipping benefit, with respect to the performances of the “deterministic” solution based on sea level forecasts alone. 100,000 Monte Carlo simulations are used to compute these statistics, with bootstrap sampling. The chance constrained and GMM or Gaussian sampling are not represented here as the reduction in guaranteed benefit is out of scope, reaching 200%.

decision in real conditions. Second, the spatial organisation of the points underlines a general pattern in robust optimisation: the guaranteed benefit increases at the cost of the increase in variance $\sigma^2$. Finally, as noted by [?], the variation in actual benefit is about one order of magnitude smaller than the reduction in its standard deviation.

5 Conclusion

Figure ?? summarises some of the above considerations in a 3-dimensional view of the optimisation problem. A map of the standard deviation is estimated with bootstrap sampling for each couple $(t, m)$ of the search space. On top of the map, we report the decision suggested by the net benefit optimisation from sea level forecasts, perfect forecasts (i.e., perfect knowledge of the future) and by four optimisation approaches. Figure ?? gives a good overview of the set of solutions returned by all the approaches and presented above.

As the owner of the company, you could use the benefit optimisation decision that is based on the deterministic harmonic forecasts, load 3,835 mt of barley and cast off at 23:00. However, the outcome of this decision, given the actual observations of sea levels is $-2.15B_0$. This is much less desirable than the benefit $2.12B_0$ that you could make if you knew the future perfectly and left Lowestoft port at 00:30 with 4,362 mt on board. Using the stochastic optimisation method developed in this paper, you could load cargo between 3,935 and 3,959 mt, raise anchor between 00:30 and 01:00 and get a net benefit from $1.89B_0$ to $1.91B_0$. If these decisions were reported in Figure ?? (mapping based on actual sea level conditions), one could notice that a port re-scheduling of up to 2 hours (earlier) or 4 hours (delay) would not substantially change the benefit, nor a variation (in standard limits) in sea level conditions. Besides, Figure ?? reminds that the variance in the actual benefit is substantially reduced for our solutions, contrary to the variance of the deterministic proposition. In other words, the approach $R_{PSO}$ proposes a robust solution. This is true for any risk metric introduced here apart from the chance-constrained, and true for any sampling distribution although a Gaussian generally leads to solutions with less predictable economic outcomes. Recalling the questions raised in the motivation of the problem (Section ??), in this case study, our stochastic approach demonstrated to be economically valuable with respect to the standard (deterministic) approach. Besides, a simple Logistic modelling of the residuals is enough to produce quality results, similar to those gained by means of a GMM.

One can note that the cargo load output $m^*$ can be turned into a safety margin $\Delta r$ to be deducted
Figure 3: Three dimensional mapping of each decision \((t_d, m)\) to the associated actual benefit standard deviation. Points of interest discussed in the text are also reported. The mapping use Monte Carlo simulations of 1,000 journeys by means of bootstrap re-sampling.

from the maximum draft that would have been allowed given the sea level tide forecasts at hand at \(t_0\) (procedure \(B_{PSO}\)). For future works, it would be interesting to compare \(\Delta r\) with what “non-stochastic” commercial softwares would suggest on a similar problem, so as to assess the quality and potential added value of our model.

Avenues of research on the problem raised in this paper include defining sounder uncertainty sets on which the risk metrics would then be applied. A finer modelling of the sea level residuals would also be judicious, exploiting the cyclic character of data.

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